Remarks on the Modeling of Fluidized Systems

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In the equations governing the flow of fluidized systems, one often comes across a modulus $G(\epsilon)$, referred to as the modulus of elasticity. The meaning and relevance of this modulus have been the subject of considerable discussion in the field. In this work, we would like to address the issue that values used for $G(\epsilon)$ are orders of magnitude apart (for detail, see Massoudi et al., 1992) and point out that such a modulus, derived and used in the manner of Ettehadieh et al. (1984) and Gidaspow (1986), need not be central to the theory of fluidization. An alternative approach is presented by Johnson et al. (1991).

We discuss only the balance of momentum for the solid in the y direction used by Gidaspow and his coworkers (1983, 1985, 1986a,b,c, 1987, 1989) (the complete set of equations can be found in Gidaspow, 1986) for the sake of brevity, since our discussion is on the consequences of this equation:

$$\frac{\partial}{\partial t}[\rho_s(1-\epsilon)v_s]+\frac{\partial}{\partial x}[\rho_s(1-\epsilon)u_s v_s]+\frac{\partial}{\partial y}[\rho_s(1-\epsilon)v_s v_s]$$
$$=-(1-\epsilon)\frac{\partial p}{\partial y}+\beta_y(v_g-v_s)-\rho_s(1-\epsilon)g-\frac{\partial \tau}{\partial y}, \quad (1)$$

where τ is the solid stress. Gidaspow and his coworkers assume the constitutive equation for the solid stress τ of the form:

$$\tau=\tau(\epsilon). \quad (2)$$

In most of these studies, the viscosity of the fluid only enters through the interaction term for the drag, but the divergence of the stretching tensor $\boldsymbol{D}$, which accounts for the Δv in the equations of motion, is left out. Furthermore, the granular solids are assumed to behave like a fluid, and there appears a spherical part of the stress denoted by p_s that is usually referred to as the solid pressure. Frequently it is assumed that this term is the same as the fluid pressure or is related to it in some manner, for otherwise we have a closure problem (cf. Johnson et al., 1991): there are more unknowns than equations. Applying the chain rule, Gidaspow (1986) obtains:

$$\frac{\partial \tau}{\partial y}=\frac{\partial \tau}{\partial \epsilon}\frac{\partial \epsilon}{\partial y} \quad (3)$$

where the y-direction is defined as the vertical direction. The particle-to-particle interaction coefficient (referred to as the modulus of elasticity) is defined as:

$$G(\epsilon)=\frac{\partial \tau}{\partial \epsilon} \quad (4)$$

where this relationship would now have to be determined using an experiment. Gidaspow (1986) discusses the importance of this term: "This term becomes of numerical significance only when the void fractions go below the minimum fluidization void fraction. It also helps to make the system numerically stable, because it converts the imaginary characteristics into real values. For some calculations, it was necessary to adjust this stress to prevent the void fraction from reaching impossibly low values."

Based on the experimental results of Rietema and Mutsers (1973), Gidaspow obtains an empirical relation for $G(\epsilon)$. Gidaspow also uses various other correlations for $G(\epsilon)$ and the forms given by Gidaspow and coworkers are shown in Table 1. Even a cursory glance at the table reveals the wide range of

Table 1. Various Correlations for $G(\epsilon)$

Authors	Form of $G(\epsilon)$
Gidaspow & Ettehadieh (1983)	$G(\epsilon)=-10^{-8.76\epsilon+5.43}$ N/m^2
Ettehadieh et al. (1984)	$G(\epsilon)=-10^{-10.46\epsilon+6.577}$ N/m^2
Shih et al. (1987)	$G(\epsilon)=-10^{-9.76\epsilon_1+7.8}$ N/m^2 $G(k)=\epsilon_k\, G(\epsilon_1)$ for $k=2, \ldots, n$ "Multiparticle Systems"
Syamlal and O'Brien (1988)	$G(\epsilon_1)=1.5\times10^{-3}\, e^{500(0.4-\epsilon_1)}$ $\nabla P_s=\epsilon_k\, G(\epsilon_1)\ \nabla\epsilon_1$ "Multiparticle Systems"
Gidaspow et al. (1989)	$G(\epsilon)=-10^{-10.5\epsilon+9.0}$ N/m^2
Bouillard et al. (1989)	$G(\epsilon_s)=G_o\, e^{-c(\epsilon-\epsilon^*)}$ $c=600$ $\epsilon^*=0.376$ $G_o=1.0$ Pa
Aldis and Gidaspow (1989)	$G(\epsilon)=-e^{a(1-\epsilon)}$ $a=$ constant

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values ascribed to $G(\epsilon)$. In some articles, Gidaspow and his coworkers make it clear that they use the data for glass beads. However, Ettehadieh et al. (1984) used two values for $G(\epsilon)$ that are orders of magnitude apart. Gidaspow (1986) also used a value for $G(\epsilon)$ that corresponds to a value for glass beads at a value of void fraction of 0.8. The results of Rietema and Mutsers (1973), however, have data only up to $\epsilon_{max}=0.55$, where ϵ_{max} is the maximum void fraction for glass. Rietema and Mutsers (1973) (and the corrected results of Mutsers and Rietema, 1977) do have values for $G(\epsilon)$ for $\epsilon_{max}=0.8$, which is however, far different from the values used by Gidaspow.

These different forms used for $G(\epsilon)$ are not all based on the experimental results of Rietema and Mutsers (1973). The theory, however, seems to be insensitive to the value of $G(\epsilon)$ even though the term $G(\epsilon)$ multiplies the highest order derivative of the void fraction. The claim that such a term is necessary for the system to be numerically stable is not compelling enough to have it in the theory, more so when various forms for $G(\epsilon)$ produce the same numerical result (Gidaspow, 1990).

We have used the original data of Rietema and Mutsers (1973), which contains the experimental error to obtain the curve fit. Mutsers and Rietema (1977) explain the source of error as: "In earlier publications (Rietema, 1973; Rietema and Mutsers, 1973), higher values of d_p [and therefore also of $G(\epsilon)$] were given for polypropylene. Agglomeration occurring during the Coulter counter analyses was found to be the cause of this error." The curve fitting is performed both by hand and an IMSL subroutine, which uses least squares. The results show that for polypropylene,

$$G(\epsilon) = -10^{-13.11\epsilon+9.90}\ \text{N/m}^2 \tag{5}$$

which is not the same as the values reported in Table 1. Next, we used the data of Mutsers and Rietema (1977), where the error had been corrected. The curve fitting shows that the corrected value of "the modulus of elasticity" $G(\epsilon)$ for polypropylene is:

$$G(\epsilon) = -10^{-23.39\epsilon+16.06}\ \text{N/m}^2 \tag{6}$$

which is also very much different from the values in Table 1. The values assumed by Gidaspow and coworkers for $G(\epsilon)$ does not agree with either the original experimental data of Rietema and Mutsers (1973) or the later corrected version of Mutsers and Rietema (1977). For example, if we use the $G(\epsilon)$ given by Gidaspow (1986):

$$G(\epsilon) = -10^{-8.76\epsilon+5.43}\ \text{N/m}^2 \tag{7}$$

and assume a value of 0.4 for ϵ (almost at minimum fluidization), we obtain:

$$G(\epsilon) = -84.3\ \text{N/m}^2 \tag{8}$$

Now for the same ϵ, Eq. 6 gives:

$$G(\epsilon) = -5.06\times10^{6}\ \text{N/m}^2 \tag{9}$$

which is orders of magnitude larger.

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Notation

g = gravity
p = pressure
u_s, v_s = solid velocities in x and y direction
v_g = gas velocity
β = coefficient of drag
τ = solid stress
ϵ = volume fraction
ρ_s = density of solid particles
$G(\epsilon)$ = modulus of elasticity

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